Chaotic Behavior in a Relativistic Electron Beam Interacting With a Transverse Slow Electromagnetic Wave

A. Serbeto[1] and M. Virgínia Alves[2]
[1] Instituto de Física - Universidade Federal Fluminense
Outeiro de São João Batista s/n, 24020-004 Centro, Niterói, RJ, Brasil
[2] Laboratório Associado de Plasma - Instituto Nacional de Pesquisas Espaciais
12227-010, São José dos Campos, SP, Brasil

Abstract

Using a nonlinear set of equations which describes the excitation of a purely tranverse slow electromagnetic wave by a relativistic electron beam we have shown that the system runs from chaotic behavior to a regular stable state due to crisis phenomenum and from stabilized soliton and repeated stabilized explosive solutions to a temporal chaos.These behaviors suggest that the primary mechanism for the saturation of the explosive instability is not only the cubic nonlinear frequency shift as pointed out by many authors until now.The inclusion of the the velocity perturbation in the beam charge initial equilibrium state leads the system to these strange behaviors.

1 - INTRODUCTION

The generation of high-power coherent radiation using a straight relativistic electron beam in dielectric-filled waveguides has attracted considerable interest, recently. The devices based on this effect, such as the high efficiency Cerenkov oscillators and amplifiers, have been a persisting goal in the scientific community since very high frequency oscillations can be obtained using a relativistic or even nonrelativistic electron beam[1,2]. The dynamics of particles and fields can be described statistically by a group of sample electrons through the Hamilton-Jacobi and Maxwell equations, kinetic Vlasov theory or by collectively through the fluid model. These descriptions provide the main relevant properties of the nonlinear effects due to the interaction of a free-electrons with electromagnetic fields. However, present theoretical results do not fully cover all the possible regimes of the stimulated scattering and mechanism of their nonlinear stabilization. Recently, Bogdanov, Kuzelev and Rukhadze[3] have analyzed a nonlinear mechanism for the excitation of a purely electromagnetic wave by a rectilinear electron beam involving a resonant excitation of second harmonics on the beam space-charge wave. They have shown that this excitation is explosive if the nonlinearity is quadratic. Considering the nonlinear frequency shift they pointed out that cubic nonlinearity is the primary mechanism for saturating the explosive instability. In this letter, we examine this effect, using the collective variable description for free-electron emitters proposed by Bonifacio et al.[4], on the nonlinear excitation of an electromagnetic wave interacting with a relativistic electron beam. By numerical simulation using a Runge-Kutta subroutine we have found soliton[5-8], chaos[9-12], crisis[13] solutions and, also, the well known explosive instability solution given in Ref.[3,5]. In our model these solutions demonstrate the coexistence of coherent temporal structure (soliton, explosion and repeated stabilized explosions) and temporal chaos. The chaotic behavior is diagnosed by the Fourier power spectrum and one-dimensional non-inversible return map[12], quite similar to the Lorenz return map. All these results are completely different from those described in Ref.[3]. We have found that the primary mechanism for the saturation of the explosive instability can also come from the introduction of a charge particle velocity perturbation from the initial equilibrium state. Due to the sensitivity of the dynamical behavior of the system on the control parameters and on the initial conditions, the explosive instability can saturate in a different way from that considered in the present literature. The crisis phenomenun[13], as a consequence of the small change on the control parameters, leads to an unsaturated explosive instability, which in this case will be saturated by the nonlinear frequency shift, as one expects, in agreement with Refs.[3,5].

2 - DYNAMICAL EQUATIONS

The nonlinear set of equations which describes the excitation of a purely transverse slow electromagnetic wave (EMW) by a relativistic electron beam is:

$$\frac{du_1}{d\tau} = -\frac{\nu}{4} u_1 u_2 \sin \Phi_0 , \tag{1}$$

$$\frac{du_2}{d\tau} = -\frac{u_1^2}{2\mathcal{F}(u_1)} \sin \Phi_0 , \tag{2}$$

$$\frac{d\Phi_0}{d\tau} = -\frac{1}{2u_2}\Big\{ \nu u_2^2 + \frac{u_1^2}{\mathcal{F}(u_1)} \Big\} \cos \Phi_0 + \mathcal{G}(u_1, u_2) , \tag{3}$$

where

$$\mathcal{F}(u_1) = 1 + 2\,\langle P_j \rangle_0 - \frac{4}{\nu}[u_1^2 - u_{10}^2]$$

and

$$\mathcal{G}(u_1, u_2) = -1 + \mathcal{F}(u_1) -$$

$$\frac{1}{\mathcal{F}(u_1)}\Big\{ 2\,\langle P_j^2 \rangle_0 + \frac{4}{\nu}(u_1^2 - u_{10}^2) - (u_2^2 - u_{20}^2) - 2u_1^2 u_2 \cos \Phi_0 + 2\mathcal{K}(0) \Big\}$$

are defined as "energy-momentum" functions due to the space-charge wave interacting with an electromagnetic wave. These functions play the fundamental role on the system defined above. The constant $\mathcal{K}(0)$ is defined as $u_{10}^2 u_{20} \cos \Phi_0(0)$, where u_{10}, u_{20} are the wave amplitudes at time $\tau = 0$ and $\Phi_0(\tau) = 2\phi_1 - \phi_2$ is the relative phase; u_1 and u_2 are the wave amplitude at time $\tau; \nu = \sqrt{2(\gamma_b^2 - 1)} \times (\omega_* / \omega_1), \omega_* = \omega_b^2 / \gamma_b \epsilon$, $P_j = \sqrt{2}\, ck \delta \gamma_j / \beta_b \gamma_b^3 \omega_p$ is the normalized canonical momentum, β_b is the normalized electron beam speed, γ_b is the relativistic factor, $\omega_p^2 = \omega_b^2 / \beta_b^2 \gamma_b^3 \epsilon, \omega_b^2 = 4\pi q^2 n_0 / m$ is the squared beam-plasma frequency, $\delta_j = \sqrt{2} ck \beta_b / \omega_p$ and $\tau = \omega_p t / \sqrt{2}$ is the normalized time. In the limits of $\nu \longrightarrow \infty$, $\langle P_j \rangle = 0$ and $\langle P_j^2 \rangle = 0$ the system reduces to a system of equations quite similar to equations derived in Ref.[3]. Since in real physical systems these parameters have finity values, even small ones, we can obtain different solutions of those presented in Ref.[3]. This is the situation that we have realized the numerical simulation to obtain all the necessary information about the system for different values of the control parameters.

3 - NUMERICAL RESULTS

We fix initial conditions $u_{10} = 10^{-2}, u_{20} = 10^{-23}$ chosen in order to avoid numerical singularity, $\Phi_0(0) = 0, \langle P_j \rangle_0 = 0$ and $\langle P_j^2 \rangle_0 = 0$. Using $\nu = 1.18720$ the system presents a repeated stabilized explosive solution as shown in the Fig.01-(a). For $\nu = 1.18721$ and keeping the same initial conditions for field amplitudes and beam parameters we get a stabilized soliton solution[5] as given in Fig.01-(b). For $\nu = 1.5$ and keeping the same initial conditions for field amplitudes and beam parameters we get an unsaturated explosive instability as given in Fig.01-(c). The numerical computation has shown that the increasing of ν to a threshold value for definiteness $\nu_{th} = 5.55338$ the time evolution of the system suffers many bifurcations leading it to a chaotic behavior, with unknown route at the present time. Fig.01-(d) shows the time evolution for the wave amplitude $u_1(\tau)$ for $\nu = \nu_{th}$. Fig.01-(e) shows the Fourier power spectrum of $u_1(\tau)$ for $\nu = \nu_{th}$. This picture shows a large broad band spectral bahavior typical of chaotic system. Fig.01-(f) shows the maxima points $u_1(n+1) \times u_1(n)$ ocurring during 3000 interactions for $\nu = \nu_{th}$. This Poincaré map[12], quite similar to the Lorenz map, which all maxima appear to lie on a curve, shows the existence of a strong dissipation in the system which leads to chaotic oscillations on the wave amplitudes. The intersection of the diagonal $u_1(n+1) = u_1(n)$ means that there is a fixed point for a dynamical equilibrium of the system.

Since our system is very sensitive to very small changes on the initial conditions for the field amplitudes, u_{10} and u_{20}, with relative phase $\Phi_0(0)$ and on the beam controlled parameters $\langle P_j \rangle_0$ and $\langle P_j^2 \rangle_0$, which are related to the injected beam energy and to the initial energy spread, we have further run the numerical simulation for different values of these input parameters. We observe that the dynamical behavior of the system changes from chaos to explosive solution through crisis, and so on, according to the values of these parameters. To finish the analysis we can say that the primary mechanism for saturating the instabilities on the system is not only a frequency shift, due to the cubic nonlinearity, as pointed out by many authors in the wave-wave interaction. With the inclusion of the velocity perturbation on the initial equilibrium state of the beam particles many and well defined different solutions for the system can be obtained depending on the control parameters, even if the relativistic beam is modulated on the initial state of the first harmonics. So, one can say that many computer simulation shall be done in order to well understand this dynamical system, quite sensitive to the input parameters. This will appear on future publication.

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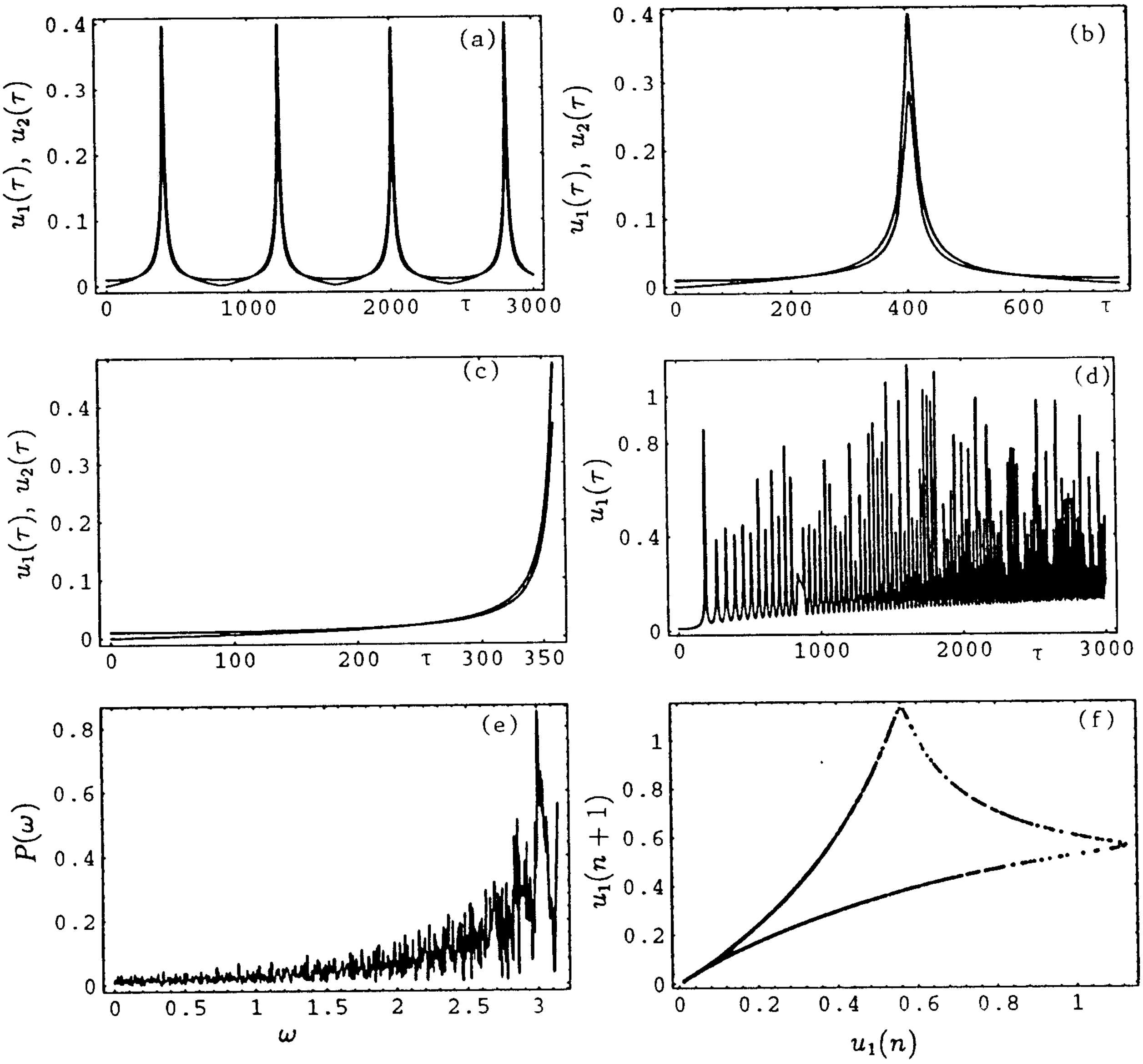

Figure 1: Numerical solutions of Eqs. (1)-(3). We fix initial conditions $u_{10} = 10^{-2}, u_{20} = 10^{-23}$ chosen in order to avoid numerical singularity, $\Phi_0(0) = 0, \langle P_j \rangle_0 = 0$ and $\langle P_j^2 \rangle_0 = 0$. (a) for $\nu = 1.18720$ the system presents a repeated stabilized explosive solution; (b) for $\nu = 1.18721$ the system presents a stabilized soliton solution; (c) the time evolution for the wave amplitude $u_1(\tau)$ for $\nu = 5.55338$; (e) the Fourier power spectrum of $u_1(\tau)$ for $\nu = 5.55338$; (f) for $\nu = 5.55338$, the maxima points $u_1(n+1) \times u_1(n)$ ocurring during 3000 interactions. In the Fig.01-(a)-(c), the curve that describes the time evolution of the wave amplitude $u_1(\tau)$ starts at 10^{-2}. The second curve describes the time evolution of the wave amplitude $u_2(\tau)$.

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Short–period Wiggler FEL Design Using a R. F. Accelerator as a Beam Energy Source

A. Serbeto
Instituto de Física
Universidade Federal Fluminense
Outeiro de São João Batista s/n
24020 Centro, Niterói, Rio de Janeiro, Brasil

A. C. J. Paes, C. R. S. Stopa, A. C. Migliano and F. Sircilli
Instituto de Estudos Avançados–CTA
12231-970, São José dos Campos, São Paulo, Brasil

Abstract: *A nonlinear analysis for a new deigned FEL oscillator is performed to calculate the optimized efficiency of energy extraction from a relativistic electron beam generated by RF accelerator. The value of the efficiency is presented for a tapered wiggler FEL oscillator.*

High power, tunable waveguide Free-electron laser in the region of millimeter wavelength has been recently studied by many researchers, due to new dynamics features that affect the tuning capability, the mode sensitivity, and the short pulse operation at long wavelength. At the Instituto de Estudos Avancados (IEAv) we are proposing a novel design of a waveguide FEL which is of great interest in applications as isotope separation, Electron Cyclotron Resonant Heating (ECRH) of plasma in advanced magnetic fusion configuration and radar system operating at millimeter wavelength. Compared with radar at conventional longer wavelengths, superior target imaging would be obtained with enhanced ability to discriminate between different types of objects. This new designed tapered FEL operates in a Compton regime with bunched electron beam produced by R.F. accelerators. The present paper considers a free-electron configuration which aims to decrease the voltage requirement while keeping large output power and achieving a high energy extraction efficiency. This configuration presents a short period wiggler ($\lambda_\omega \sim 3.2cm$) in order to reduce the required electron beam energy in the FEL interaction. To have a strong FEL interaction in a small period wiggler, all the electrons must be at a small distance from the wiggler compared with the wiggler period.

Several issues arise in a long wavelength FEL driven by a RF accelerator as a result of the short electron bunch duration. When the bunch length is comparable to the operating wavelength λ_r, a considerable amount of coherent emission is expected even with no feedback in the resonator [1 - 4]. Since the growth of FEL pulse is strongly affected by the slippage, we can provide this strong interaction considering that the slippage length of the coherent radiation is smaller than the electron pulse length. The effect of the coupling between the radiation and particles is given by the ponderomotive wave generated by the beating of the wiggler and radiation fields. The ponderomotive wave effect is obtained in the FEL bouncing frequency which is roughly proportional to the pon-